Assembling mathematical concepts through trans-individual coordinated movements: the role of affect and sympathy

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This paper explores new developments in affect theory for studying the circulation of affect across mathematics classrooms. We use Maxine Sheets-Johnstone's term "affectivity" to characterize the responsive nature of bodies and the potential scaling-up through sympathy and coordinated movement. We examine cooperative classroom tasks that entail sympathetic coordinated movements, including diverse kinds of often imperceptible body movement (gesture, face, eye, foot, etc.). We discuss how mathematical concepts are assembled through the affective bonds that form when students participate in these tasks. Our methodology is notable for how it bridges three scales: (1) the micro-phenomenological scale of the pre-individual affect, (2) the individual scale of human movement, and (3) the transindividual scale of collective endeavour (the making of a concept).

Keywords: Affect, sympathy, collaborative tasks, movement, materiality.

Introduction

Research into the various ways that the human body factors in mathematics education has recently expanded, as new theoretical developments and innovative experimental methods have introduced significant insights about the material dimensions of teaching and learning. Some have criticized this work for how it downplays the role of the environment more broadly, while others have expressed concerns that, even as it attends more carefully to the role of the body in teaching and learning, it seems to support a mind/body split, with continued emphasis on individual cognition rather than collective and distributed learning experiences. These concerns are linked to the methodological use of design experiments that are all too often narrowly focused on individual experience. This narrow focus on the individual is particularly pronounced when we turn to research on the role of affect in mathematics education, which has typically focused on the relationship between beliefs, attitudes and emotions within individual bodies (Mcleod & Adams, 1989; Zan et al., 2006).

The socio-cultural turn tried to correct this narrow focus and looked to emotions as socially organised phenomena that are constituted in discourse and shaped by relations of power, but this approach tends to lose sight of the specific practices entailed in mathematical behaviour. Moreover, most socio-cultural studies of the emotional dimension of mathematics continue to assign particular emotions to particular students, who show frustration or anxiety or joy, as they encounter the socio-cultural rituals of school mathematics (Radford, 2015). Such an approach remains focused on the individual, rather than the affective ecology, and tends to black-box the mathematics. More recent attempts to move from beliefs to "affective systems" show promise in their attempt to study *ensembles* of emotions, feelings, attitudes, beliefs, and conceptions (Philippou & Christou, 2002),

and in their recognition that affect is dynamic and variable in intensity (Pepin & Roesken-Winter, 2015). And yet we find the research therein continues to methodologically emphasize expressions of belief and value, through reliance on conventional research methods, such as interviews and self-reporting, without actually operationalizing key ideas from systems theory (Varela & Depraz, 2005) and without tapping the extensive work outside of psychology on affective networks (Massumi, 2015). Moreover, attending to the dynamics of emotional or motivational states in a classroom or other learning community are still rare (Hannula, 2012).

In this paper we pursue a theory of affect that better helps us follow the *movement* of affect across learning events with multiple and diverse participants, in such a way that the mathematics itself is imbricated within the process. This involves delving deeper into the affective nature of mathematical practices which are lived in and through material practices. For that purpose, we turn to recent work on affect in the humanities. Since the 2000s, scholars across the humanities have pursued what is known as the *affective turn* (Clough & Haley, 2007; Gregg & Seigworth, 2010). Shifting away from psychological approaches that focus on affect as individual judgements of value (like, dislike, happy, unhappy), this new approach aims to study the *collectively dispersed nature of affect across a material ecology* (Gregg & Seigworth, 2010). In particular, we follow Massumi (2015) and Sheets-Johnstone (2009, 2011) in studying affect and emotion less as that which is produced and possessed by a psychological subject, and more as an impersonal intensive flow across relational and provisional learning assemblages. We use the term 'learning assemblage' to designate the way learning is achieved through affective resonances and the assembling of diverse agencies. The challenge is then to develop research methods that lend themselves to the study of complex ecologies of material-mathematical practices.

Our approach is significant for how it moves away from the individualistic theories of cognitive psychology towards a renewed interest in (1) the somatic and embodied expressions of affect, as bodily organic forces rather than ideational enactments of interior states and (2) the transindividual collective nature of circulating affect. The flow of affect contracts and expands across an event, recruiting our bodies and participation to varying degrees, where affect is itself a kind of pre-conscious micro-movement. The notion of "degree" is crucial here, as it underscores how affect can be contracted in one body and not another with varying intensity. This approach studies classrooms as dynamic affective ecologies and tracks the way that learning rests fundamentally on somatic and unconscious ways of moving together. Concepts emerge and settle in such an environment as a function of sympathy (de Freitas, 2018). We believe that sympathy is the seed of learning because it affords opportunities for collaborative inventive practices. We emphasize this point, because it helps open up discussion of how achievements in classrooms are truly collective insofar as they are *done through us (and not by us)*. This directs attention to the collective nature of learning.

In this paper we discuss briefly a teaching experiment to show how mathematical concepts can be coordinated through affect and sympathetic relations. In particular, we focus on the coordinated movements of two girls in a grade nine classroom, Barbara and Lucrezia, while they are working on a specific task. We track the way that the task brought forth opportunities for these two girls to develop new forms of relationality in their shared achievement, and that their coordinated

movements are directly linked to the complex set of differentials and gradients that comprise the circle concept under study. The methodology involved a teaching experiment that *primed the classroom* so that the circle concept could only be achieved through a coordination of different kinds of movement. In other words, the mathematical task demanded a sympathetic coordination between students. Our initial video analysis focused on the verbal and the gestural. A second analysis focused on micro-movements (head orientation, facial expression, rhythm and speed of coordination) which involved new coding methods that were then combined with field notes from classroom observation. Although not adequate space here to present our methods in detail, we discuss briefly how the data can be analysed in terms of affective ecologies (for more see de Freitas, Ferrara, & Ferrari, 2018).

Theoretical framework for multi-scale analysis

The words emotion and affect are commonly used together, not always with too much care for their different meanings. Here, we draw on the work of Maxine Sheets-Johnstone (2009, 2011), to help distinguish these terms, and to build a theory of affectivity that lends itself to an analysis of pre-individual and trans-individual activity. Sheets-Johnstone is at pains to show how emotions are not only "coping mechanisms" that evaluate or appraise or cope with the sudden break-down of rational discernment. She describes *affectivity* as the fundamental "responsivity" of life, drawing on a long line of phenomenology. Affectivity characterizes the way bodily activity is implicated in collective feelings (common sensibility) but also in pre-conscious sensibility. Affectivity thus characterizes the responsive pre-conscious nature of bodies, how they turn away or lean in, and at the same time how they join with other bodies in coordinated movements. For Sheets-Johnstone, there is a congruency between affect and bodily motion, precisely because affect is lived through bodily movement. In other words, the dynamics of feelings (of comfort, agony, excitement, …) coincide with micro-facial expression, minute changes in bodily posture, foot-tapping rhythms, changes in heart rate, etc. She posits that "the affective and the kinetic are clearly dynamically congruent; emotion and movement coincide" (Sheets-Johnstone, 2009, p. 377). For Sheets-Johnstone, emotions are not enacted, but *emerge in movement*. She critiques the term 'enaction' because it continues to posit an interior state that is then enacted.

And yet we note that delight, grief, remorse, etc, all move different bodies in different ways, and that one needs to reckon with that essential heterogeneity in the emotional landscape. We therefore need to extend her work to better address this heterogeneity in experience. Our theoretical approach aims to attend to the important *tensions* and indeed corporeal *incongruencies* sustained in collective endeavours. We turn to the concept of *sympathy* to better understand how distinctive and disparate movements inform the affective dimensions of learning. The word sympathy comes from ancient Greek (sumpátheia) and refers to the state of *feeling together*, derived from a composite of *fellow* and *feeling* (Schliesser, 2015). Sympathy is a complex concept with a complex history. Over the centuries, the notion of sympathy has been used to describe all sorts of activity—everything from contagious yawn catching to cosmological harmony (Brouwer, 2015). In the 19th century, work in physiology defined sympathy as the "action of sensation, the coordination of organs in the body, and the 'social principle' that allows 'fellow-feeling' to emerge in a society." (Forget, 2003, pp.

291–292). Sympathy involves an association achieved through imagination and reason (body-mind), as well as an ethical or perhaps normative action to modify one's own actions so as to *feel with* the other. Importantly, there is no uni-directional sympathy—there is always at least two different agencies engaged. Sympathy is a kind of *agreement* between bodies, when they are mutually affected by each other and sustain a tension. We caution that such agreement is not erasure of otherness, as is often the case with appeals to empathy (Schliesser, 2015). Sympathy is "something to be reckoned with, a bodily struggle"; not a matter of identification or 'putting oneself in the other's shoes' but a matter of modulating related movements—a process of *becoming other that does not erase the other* (Deleuze & Parnet, 2007, p. 53). A sympathetic coordination is not a bland alignment, nor an identification amongst parts, nor the creation of a unified homogeneous assemblage, but rather describes the assembling of heterogeneous agencies and powers. For the purposes of this paper, we suggest that sympathy involves (1) a *contagion* of feeling, (2) a *common sense* or shared sensibility, and (3) a *compassion* for the other. Below we discuss how affectivity and sympathy can help us theorize the ways that mathematical concepts are lived through embodied encounters. Our theoretical approach is meant to bring many scales together – the pre-individual affect, the individual body, the transindividual collaboration of the two girls, and finally the fanning out of affect across the whole class.

Participants and video data

The teaching experiment involved WiiGraph technology, an interactive software application that uses Wii remotes' multiple features to detect and graphically display the location of two users (a,b) as they move along life-size number lines (Nemirovsky, Bryant, & Meloney, 2012). The experiment took place in a secondary school in Northern Italy, as part of a wider study carried out during regular mathematics lessons. The study involved a class of 30 grade 9 students (aged 15-16) in activities aimed at introducing the concept of function through a graphical approach using digital technology. In this excerpt, the students (Lucrezia and Barbara) move the Wii remotes in order to create a circle graph on a screen. WiiGraph assembles the girls' collective movement as the partial derivatives of the circle. In other words, as they move their bodies, the graph captures their instantaneous speeds $\mathrm{d}b/\mathrm{d}t$ and $\mathrm{d}a/\mathrm{d}t$. The girls' speeds must be different but coordinated for the combined effect to compose a circle. The two Wii remotes must be moved with a rhythmic pattern, and indeed at related rates of changing speed, in order to achieve the effect. The movement is thus directly linked to the mathematical relationships. We focus here on how affect circulates across minute movements as the two girls coordinate their activity to explore the circle concept. There is ample evidence of disagreement (shrugging shoulders and shaking heads) as they discuss their strategy, and indeed these tensions are the important friction that sustains a sympathetic coordination. We see that the learning assemblage evolves through these tensions, when sympathy becomes a bodily struggle. A relationship of response-ability emerges through sympathetic coordinated movement. In the transcript below, R is the researcher (second author), while L and B indicate the two girls.

L: More or less like this

B: We get a thing of this kind, maybe (***B tilts her head, raises her eyebrow as she raises her hand, twists her torso and smiles***) (**1**)

L and B both look at the screen

L: For me, no… (***L giggles***)

B: Let's try

L: … cuz, when you were here, I was here (***by crossing arms, points to the two extremes***) (**2**) (***L emphasizes their difference, then slouches and shrugs a little***)

1 **2**

B: Hm hm, a little more. You've to be here, like this, pock (***B questions L's account, and further models for L, now using her two separate hands to mimic both her and L's movements. "Pock" marks the point when the second hand reaches the maximum distance***) (**3-4**)

L: But if you go fast (***L raises pitch, as though sceptical, but with humour. Then shakes her head, and offers mocking smile***)

B: Well, fast, it's up to us (***B shrugs a little, slows slightly, but continues to move both hands to-fro***) (**5-6**)

3 **4**

5 **6**

R: Can you tell me (***the two girls both turn towards R***), excuse me, please, tell us what you've decided to do, what you're deciding to do

B: We're thinking that, because she's in front of me, we stand like this, kind of, if I start here, she starts (***B points with the other hand to a middle position. She uses confident voice and L nods approvingly***) (**7**, **8**), I start here, she starts like this, when I will arrive here, she will follow me (***performs again a back and forth movement with L***) (**9**), a little, she will be there when I will be here (**10**)

L: While she goes backward (***L interjects, and nods, looking at R***)

7 **8**

9 **10**

R: Will the speed at which you move matter?

L: Yes, yes (***L confidently nods repeatedly***)

B:	We have to move at the same distance, at the same speed, and at a constant speed
L begins nodding in agreement, but interjects as though to correct B	
L:	With the same rhythm, but she starts before, and I follow her (***L reaches her hand out and flaps the air between the two girls as she speaks***)
B:	We decide the speed, but we have to move at the same speed and at a constant speed between us (***B is incorrect in this claim***). ***L begins by nodding, then shakes her head a little, indicating some mixture of dis/agreement, but says nothing***

Table 1: B and L's discussion about how to make a circle

Data analysis and discussion

In the beginning of the project, Barbara was reluctant to take part in group work: she expressed herself in long meandering statements that often confused her class mates. In the process of the teaching experiment, we noticed a serious change in Barbara's position and relationality within the class, although some students continued to dismiss her contributions. Lucrezia, in contrast, was initially silent and timid in class. She also experienced a change in her way of engaging in collective discussions, becoming more willing to intervene and express her opinion, as the experiment unfolded. The two girls came forward to join the collaborative effort of creating a circle, despite their very different ways of being in the class. We can see the way that the productive intensity of the task comes from the various contrasts or tensions that are entailed—there are two girls, each with their own life history; two orthogonal directions to be performed; two very different movements to produce the one graph. Sympathy is the coming together of these contrasts, not so one obliterates the other, but instead as an onto-creative act in which new joint learning comes forth.

The graph of the circle (eventually achieved) is a truly collaborative effect, *a doing done through the individuals (rather than by the individuals)*. The circle is made through Lucrezia and Barbara, an achievement that emerges between the cooperating agencies. This is a task that demands all three components of a sympathetic relation: (1) there is a circulation of feeling as minute facial expressions and changes in bodily posture occur, the two girls leaning in and out, attending to the micro-scale corporeal signals that circulate beneath consciousness; (2) there is a common sense or shared sensibility in the shared obligation to follow each other and work with a shared objective (the circle concept); (3) there is the compassion for the other, and the care of ensuring that others are coming along, moderating the tensions that sustain any learning assemblage. Barbara and Lucrezia are both individually eager to achieve the circle, but all too aware that this achievement depends entirely on coordinating with the independent movements of the other.

The two girls are together determined to make a circle, and there is a shared intensity while the power to lead shifts back and forth. And yet such moving-together and power-switching is successful precisely because the two girls are coordinating at the pre-individual scale of micro gestures and *petites perceptions*. The task itself has created an opportunity for shared affect and transindividual sympathy. The flow of affect recruits other student bodies by varying degrees, when the class "oohs" and "aahs" and someone says "beautiful" as the periodic functions are shown

alongside the circles. We hear the affective tone of these responses, and can track the rippling effect across the class, as the emotion fans out. Other student bodies shift in their seats, lean in and squint, as evidence of a sympathetic investment in the collective endeavour. As Massumi (2015) claims, sympathy "can reverberate across a relational field, faster than the field of conscious calculation." (p. 84). For him, this is how the micro ethnographic scale reverberates out to other scales: "it is a defining characteristic of complex environments that the extremes of scale are sensitive to each other, attuned to each other's modulations. This is what makes them oscillatory. They can perturb each other" (p. 10). Affectivity can "channel" through the individual body, reverberating out to the larger scales. *The pre-individual scale of affect can be studied for how it fuels an enveloping social-emotional space in the classroom.*

Conclusion

We stress here that the dynamic movement buried in the mathematical concept is significant. This teaching experiment helps the students grasp the many different ways in which related movements are at work in the apparently fixed and familiar figure of the circle, deepening their understanding of the geometric concept. Thus, the task itself reveals how the affective bonds of coordinated movement are inherent to the circle concept. The task itself demands that the students form assemblages in ways that are productive of collaboratively and responsibly learning together. In this case, the bodily agreement or coordination produces rich mathematical thinking—an assembling of gradients and directions that speaks directly to the circle concept and the associated periodic functions. As the students act, they also perceive these graphs on the screen. This expanded sensitivity points to the complex entanglement of affect and concept, demonstrating how innovative technologies add to our understanding of fundamental aspects of mathematics learning. The amorphous concept of circle is implicated in mathematical activity in different ways, distinctively inflected by the flow of affect between Barbara and Lucrezia. Similarly, other mathematical concepts, if considered as dynamic and variable, are embodied in different material practices (de Freitas & Ferrara, 2015; de Freitas & Sinclair, 2017). Rather than reduce all experiences of mathematics to the same emotional note, our approach attends to the nuanced or tonal differences between one experience and another. Our aim is to attend to the specific and dynamic configuration of affect that *is* mathematics in all its multiplicity.

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